Scale depedence in Sherpa

This manual describes the usage of both precalculated alternative event weights in Sherpa and the usage of the weight components written to Sherpa's `HepMC` event record [1] to carry out an a posteriori scale variation to arbitrary scales. The following is applicable to Sherpa-2.2.0 [2, 3].

1 Precalculated alternative event weights

1.1 Specifiying automatically calculated scale variations

Sherpa provides a list of precalculated alternative event weights, if asked for it. They correspond to the would-be weights had the nominal scale been multiplied by a given factor or a different, specified parton distribution function be used. The syntax to which alternative event weights Sherpa should provide is the following

```
SCALE_VARIATIONS <muR2-fac1>,<muF2-fac1>,<PDF1> <muR2-fac2>,<muF2-fac2>,<PDF2> ...;
```

Each alternative event weight is characterised through

`<muR2-fac>` a prefactor multiplying the nominal (squared) renormalisation scale

`<muF2-fac>` a prefactor multiplying the nominal (squared) factorisation scales

`<PDF>` a parton density and its accompanying α_s parametrisation.

This syntax works for all employed scale setters of Sherpa and both Sherpa's internal PDFs and PDFs interfaced through `LHAPDF5/6`.

To specify a specific member of a PDF, its number is given as an additional argument separated by a slash. Thus, `CT10nlo/38` asks for the 38th member of the `CT10nlo` PDF set. Sherpa can also be asked to perform the calculations for a full PDF set using the suffix `[all]`. Hence, `CT10nlo[all]` is equivalent to specifying all members, `CT10nlo/0`, `CT10nlo/1`, ..., `CT10nlo/52`, separately. The `[all]`-notation only works with PDFs interfaced through `LHAPDF6` [4].

The following short-hands for most practical purposes exist.

- `SCALE_VARIATIONS <muR2-fac1>,<muF2-fac1> <muR2-fac2>,<muF2-fac2> ...;`

  This short-hand notation only varies the prefactors of the nominal renormalisation and factorisation scales. The nominal PDF will be used.

- `PDF_VARIATIONS <PDF1> <PDF2> ...;`

  This short-hand notation only varies the PDFs used in calculating the alternative weights, including their respective α_s parametrisations. The renormalisation and factorisation prefactors are kept at unity.

Thus, a complete variation using the `PDF4LHC` convention would read

```
SCALE_VARIATIONS 0.25,0.25 0.25,1. 1.,0.25 1.,1. 1.,4. 4.,1. 4.,4.;
PDF_VARIATIONS CT10nlo[all] MMHT2014nlo68cl[all] NNPDF30_nlo_as_0118[all];
```

Please note again, scales are defined as squares in Sherpa. The above syntax will create $7{+}53{+}51{+}101 = 212$ additional weights for each event.

1.2 Using the entries of the `HepMC::WeightContainer`

The automatically calculcaled alternative event weights are appended to the `HepMC::WeightContainter`. Their names follow the Les Houches convention [5], i.e. are of the form `MUR<fac>_MUF<fac>_PDF<id>`. Therein, `<fac>` are the (non-quadratic) prefactors for the renormalisation and factorisation scales relative to the nominal one and `<id>` is the integer `LHAPDF` [4, 6] PDF set member identifier.

In an analysis, the histograms can then be filled with the respective weight entry. The normalisation of this observable is then

$$\langle O\rangle^{\texttt{MUR<fac>\_MUF<fac>\_PDF<id>}} = \frac{1}{N_{\text{trial}}} \sum_i^n w_i^{\texttt{MUR<fac>\_MUF<fac>\_PDF<id>}}(\Phi)\, O(\Phi) \tag{1.1}$$

where in $w_i^{\texttt{MUR<fac>\_MUF<fac>\_PDF<id>}}(\Phi)$ is the weight of the phase space configuration Φ for variation `MUR<fac> _MUF<fac>_PDF<id>` and $O(\Phi)$ is the value of the observable for Φ. $N_{\text{trial}} = \sum_i^n n_{\text{trial}\,i}$ wherefore $n_{\text{trial}\,i}$ can be extracted from the `HepMC::WeightContainer` entry `NTrial` for each event. In other words, the sum of weights needs to be rescaled by the number N_{trial}.

In case of unweighted event generation, i.e. event generation where all events carry a uniform weight, this is by default fixed such that eq. (1.1) holds. If instead a uniform weight of 1 for the nominal scale and PDF choice is desired, each event's weight should be rescaled by w_{norm}, the value of the `HepMC::WeightContainer` entry `WeightNorm`. The expectation value of an observable is then given by

$$\langle O\rangle^{\texttt{MUR<fac>\_MUF<fac>\_PDF<id>}} = \frac{w_{\text{norm}}}{N_{\text{trial}}} \sum_i^n \frac{w_i^{\texttt{MUR<fac>\_MUF<fac>\_PDF<id>}}(\Phi)}{w_{\text{norm}}}\, O(\Phi)\ . \tag{1.2}$$

Of course, all alternative event weights also must be rescaled by the same value w_{norm}, resulting in, then potentially non-uniform, event weights ~ 1. For the nominal sample, where all event weights are equal to w_{norm}, this simplifies to

$$\langle O\rangle^{\texttt{MUR1\_MUF1\_PDF<default>}} = \frac{w_{\text{norm}}}{N_{\text{trial}}} \sum_i^n O(\Phi) \;=\; \sigma_{\text{incl}}^{\texttt{MUR1\_MUF1\_PDF<default>}}\, \bar{N}_O\ , \tag{1.3}$$

where $\bar{N}_O = \frac{1}{n}\, N_O$ is the avarage and N_O the total number of events passing the definition of the observable O. While eq. (1.3) is widely used it is applicable only to events of strictly uniform weight. In all other cases, i.e. when using partially unweighted events, be it due to the specification of `Enhance_Factor` or `Enhance_Observable` or allowing events to exceed the predetermined maximum w_{norm}, the use of eq. (1.1) or (1.2) is mandatory.

2 User calculated alternative event weights

Events generated by SHERPA and written to HepMC can also be manually reweighted to any alternative renormalisation and factorisation scale choice. To this end, however, additional information, depending on the event type, is needed. Setting HEPMC_EXTENDED_WEIGHTS=1 adds the building blocks needed for such variations to the events written to file while USE_HEPMC_EXTENDED_WEIGHTS 1 needs to be specified in its analysis block if this information is needed in the HepMC events passed as objects to Rivet through the internal Rivet interface. These additional building blocks are added to the HepMC::WeightContainer as named entries, thus at least HepMC-2.06 is required.

As different event types require different methods of computing the alternative event weight the HepMC::WeightContainer contains the following entry

Variable	Name of HepMC::WeightContainer entry
N_{type}	Reweight_Type

It can take the following values, directly encoding which type of reweighting must be performed.

Value of N_{type}	Type	Present in	Cf. section
0	LO	LO, LOPS	Sec. 2.1
1	B	NLO, NLOPS, MEPS@NLO	Sec. 2.2.1, 2.2.2, ??
2	VI	NLO, NLOPS, MEPS@NLO	Sec. 2.2.1, 2.2.2, ??
4	KP	NLO, NLOPS, MEPS@NLO	Sec. 2.2.1, 2.2.2, ??
8	DADS	NLOPS, MEPS@NLO	Sec. 2.2.2, ??
16	ClusterSteps	MEPS@LO, MEPS@NLO	Sec. 2.3, ??
32	$\mathbb{H}$-Event	NLOPS, MEPS@NLO	Sec. 2.2.2, ??
64	RS	NLO	Sec. 2.2.1

The types are additive and mutliple reweighting types can be needed for one event. Only the therefor required building blocks are the present in the HepMC-WeightContainer.

A few examples:

- A MEPS@LO event has $N_{\text{type}} = 16$.
- A S–MC@NLO $\mathbb{S}$-Event, cf. Sec. 2.2.2, the reweighting type is thus $N_{\text{type}} = 15$. Hence, the event weight contains pieces that transform B-like, VI-like, KP-like and DADS-like when shifting the renormalisation and/or factorisation scales and the corresponding information is be present in the HepMC::WeightContainer.
- A S–MC@NLO $\mathbb{H}$-Event has $N_{\text{type}} = 32$.
- A MEPS@NLO $\mathbb{S}$-Event has $N_{\text{type}} = 31$.
- A MEPS@NLO $\mathbb{H}$-Event has $N_{\text{type}} = 48$.
- A fixed-order NLO virtual correction event has $N_{\text{type}} = 2$.
- A fixed-order NLO integrated subtraction event has $N_{\text{type}} = 6$.
- A fixed-order NLO real minus real subtraction event has $N_{\text{type}} = 64$.

Please note that whereas for all Reweight_Type a summation over all components computes the new weight of the given event, fixed-order NLO real minus real subtraction events, $N_{\text{type}} = 64$, also entails the summing over multiple correlated events which can be identified by carrying the same event number.

2.1 LO and LOPS

A leading order parton level calculation with Born matrix elements at $\mathcal{O}(\alpha_s^n\alpha^m y^k)$ has the following explicit renormalisation (μ_R) and factorisation ($\mu_{F,a/b}$) scale dependences:

$$\begin{aligned}\langle O\rangle^{\text{LO}} &= \int \mathrm{d}\Phi_B\ \mathrm{B}(\Phi_B;\alpha_s,\alpha,y;\mu_R,\mu_{F,a/b})\ O(\Phi_B)\\ &= \lim_{N\to\infty}\frac{1}{N_{\text{trial}}}\ \sum_i^N \mathrm{B}(\Phi_{B,i};\alpha_s,\alpha,y;\mu_R,\mu_{F,a/b})\ O(\Phi_{B,i})\end{aligned} \tag{2.1}$$

with $N_{\text{trial}} = \sum_i^N n_{\text{trial}\,i}$ and

$$\begin{aligned}\mathrm{B}(\Phi_B;\alpha_s,\alpha,y;\mu_R,\mu_{F,a/b}) &= f_a(x_a,\mu_{F,a})\ f_b(x_b,\mu_{F,b})\ \mathrm{B}'(\Phi_B;\alpha_s,\alpha,y;\mu_R)\\ &= f_a(x_a,\mu_{F,a})\ f_b(x_b,\mu_{F,b})\ \alpha_s^n(\mu_R)\,\alpha^m\,y^k\ \mathrm{B}''(\Phi_B)\,,\end{aligned} \tag{2.2}$$

where for now only α_s is taken as running. Therein, B is the Born matrix element containing all couplings, symmetry and flux factors, and PDFs. The B′ and B″ are stripped of the PDFs and the couplings, respectively. Thus, changeing the scales $\mu_R \to \tilde{\mu}_R$ and $\mu_{F,a/b} \to \tilde{\mu}_{F,a/b}$ results in

$$\begin{aligned}\mathrm{B}(\Phi_B;\alpha_s,\alpha,y;\tilde{\mu}_R,\tilde{\mu}_{F,a/b}) &= f_a(x_a,\tilde{\mu}_{F,a})\ f_b(x_b,\tilde{\mu}_{F,b})\ \alpha_s^n(\tilde{\mu}_R)\,\alpha^m\,y^k\ \mathrm{B}''(\Phi_B)\\ &= f_a(x_a,\tilde{\mu}_{F,a})\ f_b(x_b,\tilde{\mu}_{F,b})\ \left(\frac{\alpha_s(\tilde{\mu}_R)}{\alpha_s(\mu_R)}\right)^n\ \mathrm{B}'(\Phi_B;\alpha_s,\alpha,y;\mu_R)\\ &= \frac{f_a(x_a,\tilde{\mu}_{F,a})}{f_a(x_a,\mu_{F,a})}\ \frac{f_b(x_b,\tilde{\mu}_{F,b})}{f_b(x_b,\mu_{F,b})}\ \left(\frac{\alpha_s(\tilde{\mu}_R)}{\alpha_s(\mu_R)}\right)^n\ \mathrm{B}(\Phi_B;\alpha_s,\alpha,y;\mu_R,\mu_{F,a/b})\end{aligned} \tag{2.3}$$

The variation can be computed from either form. In Sherpa's `HepMC` event record the second or third form can be used and respective factors can be accessed in the following way:

Variable	Name of `HepMC::WeightContainer` entry
$\mathrm{B}(\Phi_B;\alpha_s,\alpha,y;\mu_R,\mu_{F,a/b})$	`Weight`
$\mathrm{B}'(\Phi_B;\alpha_s,\alpha,y;\mu_R)$	`Reweight_B`
n_{trial}	`NTrial`
w_{norm}	`WeightNorm`
μ_R^2	`MuR2`
n	`OQCD`
$m+k$	`OEW`

The remaining information, x_a, x_b, $\mu_{F,a}^2$, $\mu_{F,b}^2$, $\alpha_s(\mu_R)$ and α, can be accessed throught the `HepMC::PDFInfo` object and `HepMC::GenEvent::alphaQCD/QED()`, respectively.

As we do not yet vary the scales in the parton shower, scale variations in LOPS proceed the same way. In LO or LOPS calculations with the `METS` scale setter care must be taken and the corresponding MEPS@LO proceedure must be used as additional terms arise.

For unweighted events $\mathrm{B}(\Phi_B;\alpha_s(\mu_R),\alpha,y;\mu_{F,a/b}) = \mathrm{B}^{\max} = w_{\text{norm}}$ uniformly for every event. Scale variations then work the very same way as for weighted events. Of course, applying eq. (2.3) then leads to a broader weight distribution. Partially unweighted events can be treated on the same footing.

2.2 NLO and NLOPS

2.2.1 NLO

A fixed-order parton next-to-leading order level calculation at $\mathcal{O}(\alpha_s^{n+1}\alpha^m y^k)$ using Catani-Seymour subtraction has the following renormalisation (μ_R) and factorisation (μ_F) scale dependences:

$$
\begin{aligned}
\langle O\rangle^{\text{NLO}} =& \int \Phi_B \bigg\{ \mathrm{B}(\Phi_B;\alpha_s,\alpha,y;\mu_R,\mu_{F,a/b}) \\
&\qquad + \mathrm{VI}(\Phi_B;\alpha_s,\alpha,y;\mu_R,\mu_{F,a/b}) \\
&\qquad + \int \mathrm{d}x'_{a/b}\mathrm{KP}(\Phi_B,x'_{a/b};\alpha_s,\alpha,y;\mu_R,\mu_{F,a/b}) \bigg\}\, O(\Phi_B) \\
&+ \int \Phi_R \bigg[\mathrm{R}(\Phi_R;\alpha_s,\alpha,y;\mu_R,\mu_{F,a/b})\, O(\Phi_R) \\
&\qquad - \sum_j \mathrm{D}_S(\Phi_{B,j}\cdot\Phi^j_{R|B};\alpha_s,\alpha,y;\mu_{R,j},\mu_{F,j,a/b})\, O(\Phi_{B,j}) \bigg] \\
=& \lim_{N\to\infty} \frac{1}{N_{\text{trial}}} \bigg\{ \sum_i^{N_B} \bigg\{ \mathrm{B}(\Phi_{B,i};\alpha_s,\alpha,y;\mu_R,\mu_{F,a/b})\, O(\Phi_{B,i}) \\
&\qquad + \mathrm{VI}(\Phi_{B,i};\alpha_s,\alpha,y;\mu_R,\mu_{F,a/b})\, O(\Phi_{B,i}) \\
&\qquad + \mathrm{KP}(\Phi_B,x'_{a/b};\alpha_s,\alpha,y;\mu_R,\mu_{F,a/b}) \bigg\}\, O(\Phi_B) \\
&\quad + \sum_i^{N_R} \bigg[\mathrm{R}(\Phi_{R_i};\alpha_s,\alpha,y;\mu_R,\mu_{F,a/b})\, O(\Phi_{R_i}) \\
&\qquad - \sum_j \mathrm{D}_S(\Phi_{B,j,i}\cdot\Phi^j_{R_i|B};\alpha_s,\alpha,y;\mu_{R,j},\mu_{F,j,a/b})\, O(\Phi_{B,j,i}) \bigg] \bigg\}
\end{aligned} \tag{2.4}
$$

For now, the following is restricted to NLO QCD, ie. the Born process is calculated at $\mathcal{O}(\alpha_s^n\alpha^m y^k)$. The notation of the previous section is generalised.

While changing the scales $\mu_R \to \tilde{\mu}_R$ and $\mu_{F,i} \to \tilde{\mu}_{F,i}$ in the Born contribution was detailed in Eq. (2.3), doing so in the other pieces results in

$$
\begin{aligned}
&\mathrm{VI}(\Phi_B;\alpha_s,\alpha,y;\tilde{\mu}_R,\tilde{\mu}_{F,a/b}) \\
&= f_a(x_a,\tilde{\mu}_{F,a})\, f_b(x_b,\tilde{\mu}_{F,b})\, \alpha_s^{n+1}(\tilde{\mu}_R)\, \alpha^m\, y^k \left[\mathrm{VI}''(\Phi_B) + \frac{1}{\alpha_s^{n+1}(\mu_R)} \left(c_R^{(0)} l_R + \tfrac{1}{2}\, c_R^{(1)} l_R^2 \right) \right] \\
&= f_a(x_a,\tilde{\mu}_{F,a})\, f_b(x_b,\tilde{\mu}_{F,b}) \left(\frac{\alpha_s(\tilde{\mu}_R)}{\alpha_s(\mu_R)} \right)^{n+1} \left[\mathrm{VI}'(\Phi_B;\alpha_s,\alpha,y;\mu_R) + c_R^{(0)} l_R + \tfrac{1}{2}\, c_R^{(1)} l_R^2 \right] \\
&= \frac{f_a(x_a,\tilde{\mu}_{F,a})}{f_a(x_a,\mu_{F,a})} \frac{f_b(x_b,\tilde{\mu}_{F,b})}{f_b(x_b,\mu_{F,b})} \left(\frac{\alpha_s(\tilde{\mu}_R)}{\alpha_s(\mu_R)} \right)^{n+1} \bigg[\mathrm{VI}(\Phi_B;\alpha_s,\alpha,y;\mu_R,\mu_{F,a/b}) \\
&\qquad\qquad + f_a(x_a,\mu_{F,a})\, f_b(x_b,\mu_{F,b}) \left(c_R^{(0)} l_R + \tfrac{1}{2}\, c_R^{(1)} l_R^2 \right) \bigg]
\end{aligned} \tag{2.5}
$$

with coefficients $c_R^{(i)}$ and $l_R = \log\frac{\tilde{\mu}_R^2}{\mu_R^2}$, and

$$\begin{aligned}
&\mathrm{KP}(\Phi_B, x'_{a/b}; \alpha_s, \alpha, y; \tilde{\mu}_R, \tilde{\mu}_{F,a/b}) \\
&= \left(\frac{\alpha_s(\tilde{\mu}_R)}{\alpha_s(\mu_R)}\right)^{n+1} \Bigg[f_b(x_b, \tilde{\mu}_{F,b}) \left(f_a^q \tilde{c}_{F,a}^{(0)} + f_a^q(x'_a)\tilde{c}_{F,a}^{(1)} + f_a^g \tilde{c}_{F,a}^{(2)} + f_a^g(x'_a)\tilde{c}_{F,a}^{(3)} \right) \\
&\qquad\qquad + f_a(x_b, \tilde{\mu}_{F,b}) \left(f_b^q \tilde{c}_{F,b}^{(0)} + f_b^q(x'_b)\tilde{c}_{F,b}^{(1)} + f_b^g \tilde{c}_{F,b}^{(2)} + f_b^g(x'_b)\tilde{c}_{F,b}^{(3)} \right) \Bigg]
\end{aligned} \tag{2.6}$$

with $\tilde{c}_{F,a/b}^{(i)} = c_{F,a/b}^{(i)} + c_{F,a/b}^{\prime\,(i)}\, l_{f,a/b}$ for $i \in \{0, \ldots, 3\}$, $l_{f,a/b} = \log\frac{\tilde{\mu}_{F,a/b}^2}{\mu_{F,a/b}^2}$, and

$$\begin{array}{rlrl}
& \mathbf{c = q} & & \mathbf{c = g} \\
f_c^q &= f_c(x_c, \tilde{\mu}_{F,c}) & f_c^q &= \sum_q f_q(x_c, \tilde{\mu}_{F,c}) \\
f_c^q(x'_c) &= x'_c f_c(\frac{x_c}{x'_c}, \tilde{\mu}_{F,c}) & f_c^q(x'_c) &= x'_c \sum_q f_q(\frac{x_c}{x'_c}, \tilde{\mu}_{F,c}) \\
f_c^g &= f_g(x_c, \tilde{\mu}_{F,c}) & f_c^g &= f_g(x_c, \tilde{\mu}_{F,c}) \\
f_c^g(x'_c) &= x'_c f_g(\frac{x_c}{x'_c}, \tilde{\mu}_{F,c}) & f_c^g(x'_c) &= x'_c f_g(\frac{x_c}{x'_c}, \tilde{\mu}_{F,c})
\end{array} \tag{2.7}$$

for $c = a, b$, respectively. The sum over runs over all light quark flavours for gluon initial states. Other forms for the transformation of KP can be derived, keeping in mind that $c_{F,a/b}^{(i)} \propto \alpha_s^{n+1}(\mu_R)\alpha^m y^k$, but the respective expresssions are not needed here.

Again, using Sherpa's `HepMC` event record, for B, VI, R and D_S, the second can be used and respective factors can be accessed in the following way:

Variable	Name of `HepMC::WeightContainer` entry
$\mathrm{B}'(\Phi_B; \alpha_s, \alpha, y; \mu_R)$	`Reweight_B`
$\mathrm{VI}'(\Phi_B; \alpha_s, \alpha, y; \mu_R)$	`Reweight_VI`
$\mathrm{KP}'(\Phi_B; \alpha_s, \alpha, y; \mu_R)$	`Reweight_KP`
$\mathrm{R}'(\Phi_R; \alpha_s, \alpha, y; \mu_R)$	`Reweight_RS`
$\mathrm{D}'_S(\Phi_R; \alpha_s, \alpha, y; \mu_R)$	`Reweight_RS`
n_{trial}	`NTrial`
μ_R^2	`Reweight_MuR2`
μ_F^2	`Reweight_MuF2`
$n+1$	`OQCD`
$m+k$	`OEW`
$x'_{a/b}$	`Reweight_KP_x<1/2>p`
$c_R^{(i)}$	`Reweight_VI_wren_<i>`
$c_{F,a}^{(i)}$	`Reweight_KP_wfac_<i>`
$c_{F,a}^{\prime\,(i)}$	`Reweight_KP_wfac_<i+8>`
$c_{F,b}^{(i)}$	`Reweight_KP_wfac_<i+4>`
$c_{F,b}^{\prime\,(i)}$	`Reweight_KP_wfac_<i+12>`

As before, the remaining information, x_a, x_b, $\mu_{F,a}^2$, $\mu_{F,b}^2$, $\alpha_s(\mu_R)$ and α, can be accessed throught the `HepMC::PDFInfo` object and `HepMC::GenEvent::alphaQCD/QED()`, respectively.

2.2.2 NLOPS

A parton shower matched next-to-leading order level calculation at $\mathcal{O}(\alpha_s^n \alpha^m y^k)$ using Catani-Seymour subtraction and the MC@NLO technique has the following renormalisation (μ_R) and factorisation (μ_F) scale dependences:

$$
\begin{aligned}
\langle O\rangle^{\text{NLO}} =& \int \Phi_B \Bigg\{ \mathrm{B}(\Phi_B;\alpha_s,\alpha,y;\mu_R,\mu_{F,a/b}) \\
&\quad + \mathrm{VI}(\Phi_B;\alpha_s,\alpha,y;\mu_R,\mu_{F,a/b}) \\
&\quad + \int \mathrm{d}x'_{a/b}\mathrm{KP}(\Phi_B,x'_{a/b};\alpha_s,\alpha,y;\mu_R,\mu_{F,a/b}) \\
&\quad + \sum_i \int \mathrm{d}\Phi^i_{R|B} \Big[\mathrm{D}_{A,i}(\Phi_B\cdot\Phi^i_{R|B};\alpha_s,\alpha,y;\mu_{R,i},\mu_{F,i,a/b}) \\
&\qquad\qquad - \mathrm{D}_{S,i}(\Phi_B\cdot\Phi^i_{R|B};\alpha_s,\alpha,y;\mu_{R,i},\mu_{F,i,a/b}) \Big] \Bigg\} f_B^{\text{Mc@Nlo}}(O) \\
&+ \int \Phi_R \Bigg[\mathrm{R}(\Phi_R;\alpha_s,\alpha,y;\mu_R,\mu_{F,a/b}) \\
&\qquad - \sum_j \mathrm{D}_A(\Phi_{B,j}\cdot\Phi^j_{R|B};\alpha_s,\alpha,y;\mu_{R,j},\mu_{F,j,a/b}) \Bigg] f_R(O) \\
=& \int \Phi_B \Bigg\{ \mathrm{B}(\Phi_B;\alpha_s,\alpha,y;\mu_R,\mu_{F,a/b}) \\
&\quad + \mathrm{VI}(\Phi_B;\alpha_s,\alpha,y;\mu_R,\mu_{F,a/b}) \\
&\quad + \int \mathrm{d}x'_{a/b}\mathrm{KP}(\Phi_B,x'_{a/b};\alpha_s,\alpha,y;\mu_R,\mu_{F,a/b}) \\
&\quad + \sum_j \int \mathrm{d}\Phi^j_{R|B} \Big[\mathrm{D}_A(\Phi_B\cdot\Phi^i_{R|B};\alpha_s,\alpha,y;\mu_{R,i},\mu_{F,i,a/b}) \\
&\qquad\qquad - \mathrm{D}_S(\Phi_B\cdot\Phi^i_{R|B};\alpha_s,\alpha,y;\mu_{R,i},\mu_{F,i,a/b}) \Big]_i \Bigg\} f_B^{\text{Mc@Nlo}}(O) \\
&+ \int \Phi_R\, \mathbb{H}(\Phi_R;\alpha_s,\alpha,y;\{\mu_{R,i}\},\{\mu_{F,i,a/b}\})\, f_R(O) \\
=& \int \Phi_B\, \overline{\mathrm{B}}(\Phi_B;\alpha_s,\alpha,y;\mu_R,\mu_{F,a/b})\, f_B^{\text{Mc@Nlo}}(O) \\
&+ \int \Phi_R\, \mathbb{H}(\Phi_R;\alpha_s,\alpha,y;\{\mu_{R,i}\},\{\mu_{F,i,a/b}\})\, f_R(O) \\
=& \lim_{N\to\infty} \frac{1}{N_{\text{trial}}} \Bigg\{ \sum_i^{N_B} \overline{\mathrm{B}}(\Phi_{B,i};\alpha_s,\alpha,y;\mu_R,\mu_{F,a/b})\, f_B^{\text{Mc@Nlo}}(O) \\
&\qquad + \sum_i^{N_R} \mathbb{H}(\Phi_R;\alpha_s,\alpha,y;\{\mu_{R,i}\},\{\mu_{F,i,a/b}\})\, f_R(O) \Bigg\}
\end{aligned}
\tag{2.8}
$$

wherein $f_B^{\text{Mc@Nlo}}(O)$ is the Mc@Nlo shower on Φ_B, whose scales are not included in the scale variation. Similarly, the scales of the standard shower $f_R(O)$, effected on Φ_R, is not varied.

The variations of B, VI and KP proceeds as before. The $[\mathrm{D}_A - \mathrm{D}_S]_i$ terms also transform as the B, ie.

$$
\begin{aligned}
&[\mathrm{D}_A-\mathrm{D}_S]_i\,(\Phi_B\cdot\Phi^i_{R|B};\alpha_s,\alpha,y;\mu_{R,i},\mu_{F,i,a/b}) \\
&= f_a(x_a,\tilde\mu_{F,a})\; f_b(x_b,\tilde\mu_{F,b})\; \alpha_s^{n+1}(\tilde\mu_R)\alpha^m y^k\;\; [\mathrm{D}''_A-\mathrm{D}''_S]_i\,(\Phi_B\cdot\Phi^i_{R|B}) \\
&= f_a(x_a,\tilde\mu_{F,a})\; f_b(x_b,\tilde\mu_{F,b})\; \left(\frac{\alpha_s(\tilde\mu_R)}{\alpha_s(\mu_R)}\right)^{n+1} [\mathrm{D}'_A-\mathrm{D}'_S]_i\,(\Phi_B\cdot\Phi^i_{R|B};\alpha_s,\alpha,y;\mu_R) \\
&= \frac{f_a(x_a,\tilde\mu_{F,a})}{f_a(x_a,\mu_{F,a})}\,\frac{f_b(x_b,\tilde\mu_{F,b})}{f_b(x_b,\mu_{F,b})}\,\left(\frac{\alpha_s(\tilde\mu_R)}{\alpha_s(\mu_R)}\right)^{n+1} [\mathrm{D}_A-\mathrm{D}_S]_i\,(\Phi_B\cdot\Phi^i_{R|B};\alpha_s,\alpha,y;\mu_R,\mu_{F,a/b})
\end{aligned}
\tag{2.9}
$$

It has to be noted that the partonic $x_{a/b}$ of the $[\mathrm{D}_A - \mathrm{D}_S]$ terms are those of the $\Phi_{R,i} = \Phi_B \cdot \Phi^i_{R|B}$ phase space, as are the initial state flavours a and b.

Their additional information can accessed in the following way:

Variable	Name of HepMC::WeightContainer entry
$N_{[\mathrm{D}'_A-\mathrm{D}'_S]}$	Reweight_DADS_N
$[\mathrm{D}'_A - \mathrm{D}'_S]_i(\Phi_B \cdot \Phi^i_{R\|B}; \alpha_s, \alpha, y; \mu_R)$	Reweight_DADS_<i>_Weight
$x_{a/b}$	Reweight_DADS_<i>_x<1/2>
a/b	Reweight_DADS_<i>_fl<1/2>

Therein, $N_{[\mathrm{D}'_A-\mathrm{D}'_S]}$ denotes the number of $[\mathrm{D}'_A - \mathrm{D}'_S]_i$ terms. Please note that Reweight_DADS_<i>_x<1/2>, Reweight_DADS_<i>_fl<1/2>, Reweight_DADS_<i>_MuR2, Reweight_DADS_<i>_MuF12 and Reweight_DADS_<i>_MuF22 are only present if Reweight_DADS_<i>_Weight takes a non-zero value.

The hard correction event

$$\begin{aligned}&\mathbb{H}(\Phi_R; \alpha_s, \alpha, y; \{\mu_{R,i}\}, \{\mu_{F,i,a/b}\})\; O(\Phi_R)\\ &= \Big[\mathrm{R}(\Phi_R; \alpha_s, \alpha, y; \mu_R, \mu_{F,a/b}) - \sum_i \mathrm{D}_{A,j}(\Phi_{B,j,i} \cdot \Phi^j_{R|B,i}; \alpha_s, \alpha, y; \mu_{R,j}, \mu_{F,j,a/b})\Big]\; O(\Phi_R)\end{aligned} \tag{2.10}$$

is a multiscale quantity whose components transform as in the leading order case. Its information can be accessed as in the Lo/LoPs case of Sec. 2.1, ie.

Variable	Name of HepMC::WeightContainer entry
$\mathbb{H}(\Phi_R; \alpha_s, \alpha, y; \mu_R, \mu_{F,a/b})$	Weight
$\mathbb{H}'(\Phi_R; \alpha_s, \alpha, y; \mu_R)$	Reweight_B
N_{RD_A}	Reweight_RDA_N
$\mathrm{D}'_{A,i}(\Phi_R; \alpha_s, \alpha, y; \mu_{R,j})$	Reweight_RDA_<i>_Weight
$\mu^2_{R,i}$	Reweight_RDA_<i>_MuR2
$\mu^2_{F,i,a}$	Reweight_RDA_<i>_MuF12
$\mu^2_{F,i,b}$	Reweight_RDA_<i>_MuF22
$i,\ j,\ k$	Reweight_RDA_<i>_Dipole
$\mathrm{R}'(\Phi_R; \alpha_s, \alpha, y; \mu_{R,\mathrm{real}})$	Reweight_RDA_<$N_{\mathrm{RD}_A} - 1$>_Weight
$\mu^2_{R,\mathrm{real}}$	Reweight_RDA_<$N_{\mathrm{RD}_A} - 1$>_MuR2
$\mu^2_{F,\mathrm{real},a}$	Reweight_RDA_<$N_{\mathrm{RD}_A} - 1$>_MuF12
$\mu^2_{F,\mathrm{real},b}$	Reweight_RDA_<$N_{\mathrm{RD}_A} - 1$>_MuF22
n_{trial}	NTrial
w_{norm}	WeightNorm
n	OQCD
$m + k$	OEW

Please note that value of the real emission quantities are in the last entry. The remaining information, x_a, x_b, $\alpha_s(\mu_R)$ and α, can be accessed throught the HepMC::PDFInfo object and HepMC::GenEvent::alphaQCD/QED(), respectively. Please note, that as all components share the same phase space point Φ_R, their external momentum fractions $x_{a/b}$ and flavours a/b are identical.

The dipole $\mathrm{D}_{A,j}$'s emitter (i), emitted (j) and spectator (k) indices are encoded in Reweight_RDA_<j>_Dipole in the form $10000 \cdot i + 100 \cdot j + k$, defining the mapping from the kinematics of the $\mathbb{H}$-Event to each constituent $\mathrm{D}_{A,j}$ term uniquely. This information is necessary only if a different functional form of the scales $\tilde{\mu}_R$ or $\tilde{\mu}_{F,a/b}$ is considered. The constituent R term also possesses a corresponding entry with value zero.

2.3 MEPS@LO

A leading order (QCD) multijet merged calculation with Born matrix elements at $\mathcal{O}(\alpha_s^{n+j}\alpha^m y^k)$ has the following explicit renormalisation (μ_R) and factorisation ($\mu_{F,a/b}$) scale dependences:

$$\langle O\rangle^{\text{MEPS@LO}} = \sum_{j=0}^{j_{\max}} \int \mathrm{d}\Phi_j\; \mathrm{B}_j(\Phi_j;\alpha_s,\alpha,y;\mu_R,\mu_{F,a/b},\{a_i/b_i,x_{a,i}/x_{b,i},t_i\})\; f_j(O)\,, \tag{2.11}$$

wherein j is the parton multiplicity in addition to the core process, and $f_j(O)$ is the vetoed truncated parton shower implementing both the Sudakov factors determining the survival probability of the given state Φ_j down to the merging cut and its ensuing parton shower evolution. Please note that the dependence of both the matrix element B_j and the vetoed truncated shower $f_j(O)$ on the merging cut and the resummation scale are left implicit as their uncertainty cannot presently be assessed by a simple reweighting. The tupel $\{a_i/b_i,x_{a,i}/x_{b,i},t_i\}$ (cluster step) denotes that part of the cluster history that is related to the initial state particles and therefore to the applied PDF ratios: the a_i/b_i are the possibly changing initial state flavours, the $x_{a,i}/x_{b,i}$ their momentum fractions, and the t_i are the reconstructed values of the parton shower evolution variable at each splitting. As at each branching a PDF ratio $f_c(x,t_i)/f_c(x,t_{i-1})$ is applied in the parton shower, with $t_0=\mu_{F,a}=\mu_{F,b}$ defined on the core process, the same is done in the multijet merging approach. Similarly, $x_{a,0}/x_{b,0}$ are the partonic momentum fractions of the core process.

The scales of each single α_s are determined by the cluster history. Thus, the effective global renormalisation scale is defined through

$$\alpha_s^{n+j}(\mu_R) = \alpha_s^n(\mu_R^{\text{core}}) \prod_{i=1}^{j} \alpha_s(t_i)\,. \tag{2.12}$$

For now, it is μ_R on the left hand side which is consequently varied to assess the renormalisation scale uncertainty.

Inclusive clustering – ordered histories

Inclusive cluster sequences interprete a given configuration Φ_j as a series of $1\to 2$ splittings connecting it to a core configuration which it originated from. These splittings can be all (quasi-)singular splittings present in the given model and are determined probabilisticly as an inversion of a parton shower. The resulting sequence of cluster steps $\{a_i/b_i,x_{a,i}/x_{b,i},t_i\}$ then may be either ordered or unordered.

Ordered histories are histories which satisfy $t_j<t_{j-1}<\ldots<t_1<t_0=\mu_{F,a/b}$. The constituting Born matrix elements now have the following renormalisation (μ_R) and factorisation ($\mu_{F,a/b}=t_0$) scale depenedences:

$$\begin{aligned}
&\mathrm{B}_j(\Phi_j;\alpha_s,\alpha,y;\mu_R,\mu_{F,a/b},\{a_i/b_i,x_{a,i}/x_{b,i},t_i\})\\
&= f_{a_j}(x_{a,j},\mu_{F,a})\,\frac{f_{a_j}(x_{a,j},t_j)}{f_{a_j}(x_{a,j},\mu_{F,a})}\prod_{i=0}^{j-1}\frac{f_{a_i}(x_{a,i},t_i)}{f_{a_i}(x_{a,i},t_{i+1})}\; f_{b_j}(x_{b,j},\mu_{F,b})\,\frac{f_{b_j}(x_{b,j},t_j)}{f_{b_j}(x_{b,j},\mu_{F,b})}\prod_{i=0}^{j-1}\frac{f_{b_i}(x_{b,i},t_i)}{f_{b_i}(x_{b,i},t_{i+1})}\\
&\quad\times\;\mathrm{B}'_j(\Phi_j;\alpha_s,\alpha,y;\mu_R)\\
&= \prod_{i=1}^{j}\frac{f_{a_i}(x_{a,i},t_i)}{f_{a_{i-1}}(x_{a,i-1},t_i)}\; f_{a_0}(x_{a,0},\mu_{F,a})\prod_{i=1}^{j}\frac{f_{b_i}(x_{b,i},t_i)}{f_{b_{i-1}}(x_{b,i-1},t_i)}\; f_{b_0}(x_{b,0},\mu_{F,b})\;\mathrm{B}'_j(\Phi_j;\alpha_s,\alpha,y;\mu_R)
\end{aligned} \tag{2.13}$$

While the first line reflects the implementation in SHERPA the second one best describes the physical interpretation: starting from the core process with $\{a_0/b_0,x_{a,0}/x_{b,0},t_0=\mu_{F,a/b}\}$, this state is evolved outwards to $\{a_j/b_j,x_{a,j}/x_{b,j},t_j\}$ of the phase space configuration Φ_j. In principle, with every ratio of PDFs there is also a ratio of flux factors. However, all such factors cancel except for the outermost ones, corresponding to Φ_j and, hence, are regarded as part of B''_j, B'_j and B_j. Thus,

$$\begin{aligned}
&\mathrm{B}_j(\Phi_j;\alpha_s,\alpha,y;\mu_R,\mu_{F,a/b},\{a_i/b_i,x_{a,i}/x_{b,i},t_i\})\\
&= \prod_{i=1}^{j}\frac{f_{a_i}(x_{a,i},t_i)}{f_{a_{i-1}}(x_{a,i-1},t_i)}\; f_{a_0}(x_{a,0},\mu_{F,a})\prod_{i=1}^{j}\frac{f_{b_i}(x_{b,i},t_i)}{f_{b_{i-1}}(x_{b,i-1},t_i)}\; f_{b_0}(x_{b,0},\mu_{F,b})\;\alpha_s^{n+j}(\mu_R)\,\alpha^m\,y^k\,\mathrm{B}''(\Phi_j)\,,
\end{aligned} \tag{2.14}$$

Changing the scales $\mu_R \to \tilde{\mu}_R$ and $\mu_{F,a/b} \to \tilde{\mu}_{F,a/b}$ results in

$$\begin{aligned}
&\mathrm{B}_j(\Phi_j;\alpha_s,\alpha,y;\tilde{\mu}_R,\tilde{\mu}_{F,a/b},\{a_i/b_i,x_{a,i}/x_{b,i},t_i\}) \\
&= \prod_{i=1}^{j} \frac{f_{a_i}(x_{a,i},t_i)}{f_{a_{i-1}}(x_{a,i-1},t_i)}\; f_{a_0}(x_{a,0},\tilde{\mu}_{F,a}) \prod_{i=1}^{j} \frac{f_{b_i}(x_{b,i},t_i)}{f_{b_{i-1}}(x_{b,i-1},t_i)}\; f_{b_0}(x_{b,0},\tilde{\mu}_{F,a})\; \alpha_s^{n+j}(\tilde{\mu}_R)\,\alpha^m\, y^k\; \mathrm{B}''(\Phi_j) \\
&= \prod_{i=1}^{j} \frac{f_{a_i}(x_{a,i},t_i)}{f_{a_{i-1}}(x_{a,i-1},t_i)}\; f_{a_0}(x_{a,0},\tilde{\mu}_{F,a}) \prod_{i=1}^{j} \frac{f_{b_i}(x_{b,i},t_i)}{f_{b_{i-1}}(x_{b,i-1},t_i)}\; f_{b_0}(x_{b,0},\tilde{\mu}_{F,a})\; \left(\frac{\alpha_s(\tilde{\mu}_R)}{\alpha_s(\mu_R)}\right)^{n+j} \\
&\qquad\qquad \times\; \mathrm{B}'(\Phi_j;\alpha_s,\alpha,y;\mu_R) \\
&= \frac{f_{a_0}(x_{a,0},\tilde{\mu}_{F,a})}{f_{a_0}(x_{a,0},\mu_{F,a})}\; \frac{f_{b_0}(x_{b,0},\tilde{\mu}_{F,b})}{f_{b_0}(x_{b,0},\mu_{F,b})}\; \left(\frac{\alpha_s(\tilde{\mu}_R)}{\alpha_s(\mu_R)}\right)^{n+j}\; \mathrm{B}_j(\Phi_j;\alpha_s,\alpha,y;\mu_R,\mu_{F,a/b},\{a_i/b_i,x_{a,i}/x_{b,i},t_i\})\;.
\end{aligned} \tag{2.15}$$

Please note that the clustering scales t_i are not varied as the parton shower's scales cannot be varied consistently yet. For the same reason, when changing the PDF parametrisation, only $f_{a_0}(x_{a,0},\tilde{\mu}_{F,a})$ and $f_{b_0}(x_{b,0},\tilde{\mu}_{F,b})$ are evaluated with the new PDF while all other $f_c(x,t)$ must be evaluated with the nominal PDF. Thus, the variation can be computed from either form. In Sherpa's `HepMC` event record the second or third form can be used and respective factors can be accessed in the following way:

Variable	Name of `HepMC::WeightContainer` entry
$\mathrm{B}(\Phi_B;\alpha_s,\alpha,y;\mu_R,\mu_{F,a/b})$	`Weight`
$\mathrm{B}'(\Phi_B;\alpha_s,\alpha,y;\mu_R)$	`Reweight_B`
n_{trial}	`NTrial`
w_{norm}	`WeightNorm`
μ_R^2	`MuR2`
$n+j$	`OQCD`
$m+k$	`OEW`
$N_{\{a_i/b_i,x_{a,i}/x_{b,i},t_i\}}$	`Reweight_ClusterStep_N`
t_i	`Reweight_ClusterStep_<i>_t`
$x_{a/b,i}$	`Reweight_ClusterStep_<i>_x<1/2>`
a_i/b_i	`Reweight_ClusterStep_<i>_fl<1/2>`

Please note that there are always at least two ClusterSteps present encoding the first and last ratios in the first line of eq. (2.13). The cluster sequence is thus $\{a_j/b_j, x_{a,j}/x_{b,j}, \mu_{F,a/b}\}$, $\{a_j/b_j, x_{a,j}/x_{b,j}, t_j\}$, …, $\{a_0/b_0, x_{a,0}/x_{b,0}, t_0 = \mu_{F,a/b}\}$. Hence, there are always $j+1$ ClusterStep entries present. The remaining information, x_a, x_b, $\mu_{F,a}^2$, $\mu_{F,b}^2$, $\alpha_s(\mu_R)$ and α, can be accessed throught the `HepMC::PDFInfo` object and `HepMC::GenEvent::alphaQCD/QED()`, respectively.

Inclusive clustering – unordered histories

If the whole sequence $\{t_j, t_{j-1}, \ldots, t_1, t_0 = \mu_{F,a/b}\}$ is not ordered, ie. $t_k > t_{k-1}$, then it is broken down into ordered subsequences, eg. $\{t_j, t_{j-1}, \ldots, t_k\}$ and $\{t_{k-1}, \ldots, t_1, t_0 = \mu_{F,a/b}\}$. Thus, eq. (2.13) now reads

$$\begin{aligned}
&\mathrm{B}_j(\Phi_j;\alpha_s,\alpha,y;\mu_R,\mu_{F,a/b},\{a_i/b_i,x_{a,i}/x_{b,i},t_i\}) \\
&= f_{a_j}(x_{a,j},t_j) \prod_{i=k}^{j-1} \frac{f_{a_i}(x_{a,i},t_i)}{f_{a_i}(x_{a,i},t_{i+1})}\; \prod_{i=0}^{k-2} \frac{f_{a_i}(x_{a,i},t_i)}{f_{a_i}(x_{a,i},t_{i+1})} \\
&\quad \times\; f_{b_j}(x_{b,j},t_j) \prod_{i=k}^{j-1} \frac{f_{b_i}(x_{b,i},t_i)}{f_{b_i}(x_{b,i},t_{i+1})}\; \prod_{i=0}^{k-2} \frac{f_{b_i}(x_{b,i},t_i)}{f_{b_i}(x_{b,i},t_{i+1})}\; \mathrm{B}_j'(\Phi_j;\alpha_s,\alpha,y;\mu_R)
\end{aligned} \tag{2.16}$$

in case of a single unordering at k or

$$\begin{aligned}
&\mathrm{B}_j(\Phi_j;\alpha_s,\alpha,y;\mu_R,\mu_{F,a/b},\{a_i/b_i,x_{a,i}/x_{b,i},t_i\}) \\
&= f_{a_j}(x_{a,j},t_j)\prod_{i=k_l}^{j-1}\frac{f_{a_i}(x_{a,i},t_i)}{f_{a_i}(x_{a,i},t_{i+1})}\cdots\prod_{i=k_0}^{k_1-2}\frac{f_{a_i}(x_{a,i},t_i)}{f_{a_i}(x_{a,i},t_{i+1})}\prod_{i=0}^{k_0-2}\frac{f_{a_i}(x_{a,i},t_i)}{f_{a_i}(x_{a,i},t_{i+1})} \\
&\quad\times\ f_{b_j}(x_{b,j},t_j)\prod_{i=k_l}^{j-1}\frac{f_{b_i}(x_{b,i},t_i)}{f_{b_i}(x_{b,i},t_{i+1})}\cdots\prod_{i=k_0}^{k_1-2}\frac{f_{b_i}(x_{b,i},t_i)}{f_{b_i}(x_{b,i},t_{i+1})}\prod_{i=0}^{k_0-2}\frac{f_{b_i}(x_{b,i},t_i)}{f_{b_i}(x_{b,i},t_{i+1})}\ \mathrm{B}'_j(\Phi_j;\alpha_s,\alpha,y;\mu_R)
\end{aligned} \tag{2.17}$$

in case of multiple unorderings at $\{k_0,\ldots,k_l\}$, $k_i\in\{1,\ldots,j\}$. $k_i>k_{i-1}$. In both cases, the outer ratio correcting the PDF of a_j/b_j from $\mu_{F,a/b}$ to t_j, present in eq. (2.13), has already been applied.
This transforms into

$$\begin{aligned}
&\mathrm{B}_j(\Phi_j;\alpha_s,\alpha,y;\mu_R,\mu_{F,a/b},\{a_i/b_i,x_{a,i}/x_{b,i},t_i\}) \\
&= \prod_{i=k_l+1}^{j}\frac{f_{a_i}(x_{a,i},t_i)}{f_{a_{i-1}}(x_{a,i-1},t_i)}\frac{f_{a_{k_l}}(x_{a,k_l},t_{k_l})}{f_{a_{k_l-2}}(x_{a,k_l-2},t_{k_l-1})}\cdots\prod_{i=k_0+1}^{k_1-2}\frac{f_{a_i}(x_{a,i},t_i)}{f_{a_{i-1}}(x_{a,i-1},t_i)}\frac{f_{a_{k_0}}(x_{a,k_0},t_{k_0})}{f_{a_{k_0-2}}(x_{a,k_0-2},t_{k_0-1})} \\
&\qquad\times\prod_{i=1}^{k_0-2}\frac{f_{a_i}(x_{a,i},t_i)}{f_{a_{i-1}}(x_{a,i-1},t_i)}\ f_{a_0}(x_{a,0},\mu_{F,a}) \\
&\quad\times\prod_{i=k_l+1}^{j}\frac{f_{b_i}(x_{b,i},t_i)}{f_{b_{i-1}}(x_{b,i-1},t_i)}\frac{f_{b_{k_l}}(x_{b,k_l},t_{k_l})}{f_{b_{k_l-2}}(x_{b,k_l-2},t_{k_l-1})}\cdots\prod_{i=k_0+1}^{k_1-2}\frac{f_{b_i}(x_{b,i},t_i)}{f_{b_{i-1}}(x_{b,i-1},t_i)}\frac{f_{b_{k_0}}(x_{b,k_0},t_{k_0})}{f_{b_{k_0-2}}(x_{b,k_0-2},t_{k_0-1})} \\
&\qquad\times\prod_{i=1}^{k_0-2}\frac{f_{b_i}(x_{b,i},t_i)}{f_{b_{i-1}}(x_{b,i-1},t_i)}\ f_{b_0}(x_{b,0},\mu_{F,b}) \\
&\quad\times\ \mathrm{B}'_j(\Phi_j;\alpha_s,\alpha,y;\mu_R)
\end{aligned} \tag{2.18}$$

If either only the first sequence is unordered ($k_l=j$), ie. $t_j>t_{j-1}$, or the last sequence is unordered ($k_0=1$), ie. $t_1>t_0=\mu_{F,a/b}$, the respective product is omitted. The latter leads to a vanishing factorisation scale and PDF parametrisation dependence as long as parton showering uncertainties are not included. Flux factors are nonetheless accounted for such that they cancel pairwise, as in the ordered case, necessitating them only for the external state Φ_j and collecting them in B''_j, B'_j and B_j, respectively.
Summarily, changing

$$\begin{aligned}
&\mathrm{B}_j(\Phi_j;\alpha_s,\alpha,y;\tilde\mu_R,\tilde\mu_{F,a/b},\{a_i/b_i,x_{a,i}/x_{b,i},t_i\}) \\
&= \prod_{i=k_l+1}^{j}\frac{f_{a_i}(x_{a,i},t_i)}{f_{a_{i-1}}(x_{a,i-1},t_i)}\frac{f_{a_{k_l}}(x_{a,k_l},t_{k_l})}{f_{a_{k_l-2}}(x_{a,k_l-2},t_{k_l-1})}\cdots\prod_{i=k_0+1}^{k_1-2}\frac{f_{a_i}(x_{a,i},t_i)}{f_{a_{i-1}}(x_{a,i-1},t_i)}\frac{f_{a_{k_0}}(x_{a,k_0},t_{k_0})}{f_{a_{k_0-2}}(x_{a,k_0-2},t_{k_0-1})} \\
&\qquad\times\prod_{i=1}^{k_0-2}\frac{f_{a_i}(x_{a,i},t_i)}{f_{a_{i-1}}(x_{a,i-1},t_i)}\ f_{a_0}(x_{a,0},\mu_{F,a}) \\
&\quad\times\prod_{i=k_l+1}^{j}\frac{f_{b_i}(x_{b,i},t_i)}{f_{b_{i-1}}(x_{b,i-1},t_i)}\frac{f_{b_{k_l}}(x_{b,k_l},t_{k_l})}{f_{b_{k_l-2}}(x_{b,k_l-2},t_{k_l-1})}\cdots\prod_{i=k_0+1}^{k_1-2}\frac{f_{b_i}(x_{b,i},t_i)}{f_{b_{i-1}}(x_{b,i-1},t_i)}\frac{f_{b_{k_0}}(x_{b,k_0},t_{k_0})}{f_{b_{k_0-2}}(x_{b,k_0-2},t_{k_0-1})} \\
&\qquad\times\prod_{i=1}^{k_0-2}\frac{f_{b_i}(x_{b,i},t_i)}{f_{b_{i-1}}(x_{b,i-1},t_i)}\ f_{b_0}(x_{b,0},\mu_{F,b}) \\
&\quad\times\ \alpha_s^{n+j}(\tilde\mu_R)\,\alpha^m\,y^k\ \mathrm{B}''(\Phi_j)
\end{aligned} \tag{2.19}$$

$$
\begin{aligned}
= \prod_{i=k_l+1}^{j} \frac{f_{a_i}(x_{a,i},t_i)}{f_{a_{i-1}}(x_{a,i-1},t_i)} \frac{f_{a_{k_l}}(x_{a,k_l},t_{k_l})}{f_{a_{k_l-2}}(x_{a,k_l-2},t_{k_l-1})} \cdots \prod_{i=k_0+1}^{k_1-2} \frac{f_{a_i}(x_{a,i},t_i)}{f_{a_{i-1}}(x_{a,i-1},t_i)} \frac{f_{a_{k_0}}(x_{a,k_0},t_{k_0})}{f_{a_{k_0-2}}(x_{a,k_0-2},t_{k_0-1})} \\
\times \prod_{i=1}^{k_0-2} \frac{f_{a_i}(x_{a,i},t_i)}{f_{a_{i-1}}(x_{a,i-1},t_i)} f_{a_0}(x_{a,0},\mu_{F,a}) \\
\times \prod_{i=k_l+1}^{j} \frac{f_{b_i}(x_{b,i},t_i)}{f_{b_{i-1}}(x_{b,i-1},t_i)} \frac{f_{b_{k_l}}(x_{b,k_l},t_{k_l})}{f_{b_{k_l-2}}(x_{b,k_l-2},t_{k_l-1})} \cdots \prod_{i=k_0+1}^{k_1-2} \frac{f_{b_i}(x_{b,i},t_i)}{f_{b_{i-1}}(x_{b,i-1},t_i)} \frac{f_{b_{k_0}}(x_{b,k_0},t_{k_0})}{f_{b_{k_0-2}}(x_{b,k_0-2},t_{k_0-1})} \\
\times \prod_{i=1}^{k_0-2} \frac{f_{b_i}(x_{b,i},t_i)}{f_{b_{i-1}}(x_{b,i-1},t_i)} f_{b_0}(x_{b,0},\mu_{F,b}) \\
\times \mathrm{B}'_j(\Phi_j;\alpha_s,\alpha,y;\mu_R)
\end{aligned}
$$

$$
\begin{aligned}
= \left\{ \begin{array}{cc} \frac{f_{a_0}(x_{a,0},\tilde{\mu}_{F,a})}{f_{a_0}(x_{a,0},\mu_{F,a})} \frac{f_{b_0}(x_{b,0},\tilde{\mu}_{F,b})}{f_{b_0}(x_{b,0},\mu_{F,b})} & \text{if } k_0 > 1 \\ 1 & k_0 = 1 \end{array} \right\} \left(\frac{\alpha_s(\tilde{\mu}_R)}{\alpha_s(\mu_R)}\right)^{n+j} \\
\times \mathrm{B}_j(\Phi_j;\alpha_s,\alpha,y;\mu_R,\mu_{F,a/b},\{a_i/b_i,x_{a,i}/x_{b,i},t_i\}) \,.
\end{aligned}
$$

Exclusive clustering

Exclusive cluster sequences are always ordered, involve only QCD splittings and terminate when either the specified core process is reached or an unordering is encountered. This state is then declared as new core process with its own set of core scales μ_R^{core} and $t_0 = \mu_{F,a/b}$. The reweighting proceeds then as in the ordered case of the inclusive clustering.

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